

ESTIMATES OF FARADAY ROTATION WITH PASSIVE MICROWAVE POLARIMETRY FOR MICROWAVE REMOTE SENSING OF EARTH SURFACES

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Abstract

A technique based on microwave passive polarimetry for the estimates of ionospheric Faraday rotation for microwave remote sensing of earth surfaces is described. Under the assumption of azimuth symmetry for the surfaces under investigation, it is possible to estimate the ionospheric Faraday rotation from the third Stokes parameter of microwave radiation. An error analysis shows that the Faraday rotation can be estimated with an accuracy of better than one degree with a space-based L-band system, and the residual correction errors of linearly polarized brightness temperatures can be less than 0.1 Kelvin. It is suggested that the estimated Faraday rotation angle can be further utilized to derive the ionospheric total electron content (TEC) with an accuracy of about 1 TECU= 10^{16} electrons-m⁻², which will yield 1 mm accuracy for the estimate of an ionospheric differential delay at Ku-band. Therefore this technique can potentially provide accurate estimates of ionospheric Faraday rotation, TEC and differential path delay for applications including microwave radiometry and scatterometry of ocean salinity and soil moisture as well as satellite altimetry of sea surface height. A conceptual design applicable to real-aperture and aperture-synthesis radiometers is described for the measurements of the third Stokes parameter.

1 Introduction

Ocean surface salinity and soil moisture are crucial parameters for the modeling of surface hydrological processes. Global measurements of these two parameters require the deployment of spaceborne microwave sensors. Although L-band microwave frequencies are found to be suitable for the remote sensing of ocean surface salinity and soil moisture [1, 2, 3, 4], spaceborne measurements of L-band microwave emission and radar backscatter from the earth surfaces are subjective to the influence of ionospheric Faraday rotation. The radiometric errors in the linearly polarized brightness temperatures can be larger than 10 Kelvin for incidence angles in the range of 30° to 50° . This error will limit the usefulness of data from day time satellite passes for soil moisture applications and is unacceptable for ocean surface salinity measurements, which require excellent radiometric accuracy [1, 2]. This article introduces a technique using the passive microwave polarimetry to provide accurate estimates of Faraday rotation angle for microwave remote sensing of earth surfaces from space.

Passive microwave polarimetry is not a new concept in radio astronomy [5], but is beginning to receive significant interests for earth remote sensing because of its applications for ocean surface wind velocity measurements with microwave frequencies in the range of 10-37 GHz [6, 7, 8]. Several techniques have been suggested for the measurements of the third and fourth Stokes parameters of microwave radiation, including the ferrite polarization gyrator [7], microwave polarization combiner [8], and digital correlators developed for radiometer aperture synthesis [10, 11]. The aircraft flights conducted in the 1980-90s have demonstrated that polarimetric radiometer measurements can now be made with an accuracy of better than a few tenths of Kelvin for earth surface measurements [6, 7, 8, 9]. It is proposed in this article that the polarimetric microwave measurements at about 1 GHz will provide accurate information of the ionospheric Faraday rotation, and hence a microwave instrumentation for accurate corrections of Faraday rotation is possible for space remote sensing of sea surface salinity and soil moisture.

In Section 2, we describe the principle of polarimetric radiometry for the estimates of

ionospheric rotation angle. Error estimates of this technique are described in Section 3. Section 4 discusses how to implement this technique for real aperture and synthetic aperture radiometers. Section 5 summarizes the results of this paper and discusses the issues for further investigation.

2 Polarimetric Radiometry

2.1 Stokes Parameter and Faraday Rotation

The electromagnetic waves emitted from natural media are in general partially polarized. To fully characterize the polarization state of partially polarized thermal radiation, four parameters I , Q , U , and V were introduced by Sir George Stokes [5]. These four parameters are related to the horizontally and vertically polarized components of electric fields (E_h and E_v) by the following equation:

$$I_s = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{1}{2\eta} \begin{bmatrix} \langle |E_v|^2 \rangle + \langle |E_h|^2 \rangle \\ \langle |E_v|^2 \rangle - \langle |E_h|^2 \rangle \\ 2\text{Re} \langle E_v E_h^* \rangle \\ 2\text{Im} \langle E_v E_h^* \rangle \end{bmatrix} \quad (1)$$

Here, the angular brackets denote the ensemble average of the argument, and η is the electromagnetic wave impedance [16]. The first Stokes parameter (I) represents the total radiated energy and the second Stokes parameter (Q) is the difference of the energy in the vertical and horizontal polarization channels. The third and fourth Stokes parameters U and V characterize the correlation between these two linear polarizations.

Conventional radiometers for earth remote sensing perform measurements of T_v and T_h , which are the brightness temperatures of vertical and horizontal polarizations. For convenience, we ignore a proportional constant which relates the power density to the brightness temperatures [16]. Hence, we can express T_v and T_h as

$$T_v = (I + Q)/2 \quad (2)$$

$$T_h = (I - Q)/2 \quad (3)$$

As the microwave radiation from the earth surface propagates through the ionosphere, the linearly polarized field components are rotated by an angle Ω (Faraday rotation), depending on the geomagnetic field and ionospheric electron content. It may be expressed approximately as

$$\Omega = 13557 f^{-2} N_f \langle B_0 \cos \alpha \sec \chi \rangle \quad (4)$$

where Ω is in degree, f is the radio frequency in GHz, N_f is the ionospheric total electron content (TEC) in TECU units, where 1 TECU = 10^{16} electrons-m $^{-2}$, B_0 is the earth magnetic field in Tesla, α is the angle between the magnetic field and wave propagation direction, and χ is the angle between the wave propagation direction and the vertical to the surface. The angular brackets denote an average of the enclosed quantities along the path of wave propagation. N_f is significantly affected by the solar radiation. The global TEC measurements performed by a network of GPS receivers [15] have shown significant temporal and latitudinal variations. In the equatorial and mid-latitude areas, N_f varies from a few TECU at night to as high as 60 TECU at noon, while N_f has less temporal variation with a nominal value of about 20 TECU in the polar regions. By assuming a typical worst-case geometry ($\chi = 45^\circ$, $\alpha = 0^\circ$) and a high-latitude value of $B_0 = 5.44 \times 10^{-5}$ at 300 km altitude near the peak of the electron density, an upper bound estimate for Ω in degrees at 1.4 GHz for a satellite operating above the ionosphere is [4]

$$\Omega = 0.53 N_f \quad (5)$$

Therefore, the Faraday rotation at a frequency of 1.4 GHz can be as low as a few degrees at night and as high as 20 to 30 degrees at noon local time.

Let us denote the vertically and horizontally polarized components of the electric fields received by the satellite antenna by E_{va} and E_{ha} , the linearly polarized fields after propagating through the ionosphere. E_{va} and E_{ha} will be related to E_v and E_h by

$$E_{va} = E_v \cos \Omega + E_h \sin \Omega \quad (6)$$

$$E_{ha} = -E_v \sin \Omega + E_h \cos \Omega \quad (7)$$

From the above equations, it is straightforward to show that the Stokes parameters measured by the satellite antenna are related to those emitted from the surface by

$$I_a = I \quad (8)$$

$$Q_a = Q \cos 2\Omega + U \sin 2\Omega \quad (9)$$

$$U_a = -Q \sin 2\Omega + U \cos 2\Omega \quad (10)$$

$$V_a = V \quad (11)$$

Here, the subscript “a” indicates the satellite measurements. The first and fourth Stokes parameters are insensitive to the Faraday rotation, but the second and third Stokes parameters are. It is also straightforward to show that

$$T_{va} = T_v - \Delta T_B \quad (12)$$

$$T_{ha} = T_h + \Delta T_B \quad (13)$$

where

$$\Delta T_B = Q \sin^2 \Omega - \frac{U}{2} \sin 2\Omega \quad (14)$$

ΔT_B represents the effects introduced by the Faraday rotation.

If the Faraday rotation angle Ω is known, the Stokes parameters of the surface radiation can be inverted from the satellite measurements by

$$T_v = T_{va} - Q_a \sin^2 \Omega - \frac{U_a}{2} \sin 2\Omega \quad (15)$$

$$T_h = T_{ha} + Q_a \sin^2 \Omega + \frac{U_a}{2} \sin 2\Omega \quad (16)$$

$$Q = Q_a \cos 2\Omega - U_a \sin 2\Omega \quad (17)$$

$$U = U_a \cos 2\Omega + Q_a \sin 2\Omega \quad (18)$$

Figure 1 illustrates the effects of Faraday rotation on the horizontally polarized brightness temperature and the third Stokes parameter for several values of Q . The range of Q approximately corresponds to the ocean radiation at 1.4 GHz shown in Table 1. ΔT_B can reach above 10 Kelvin at high incidence angles (high Q values) and can be lower than 1

Kelvin at near nadir angles (low Q values). The third Stokes parameter is shown to be highly sensitive to the Faraday rotation and can reach as high as about 30 to 50 Kelvin. Note that the results illustrated are applicable to land surfaces with similar Q values since ΔT_B and U are primarily modulated by the Faraday rotation with a amplitude of Q .

2.2 Azimuthal Symmetry and Estimates of Faraday Rotation

If the surfaces are azimuthally symmetric with no preferred direction of orientation, the third Stokes parameter U is zero [14]. There are non-azimuthally symmetric surfaces, such as agricultural plow fields and ocean surfaces, which have a preferred direction of corrugation. For agricultural plow fields, we are not aware of any measurements for the third Stokes parameter made at L-band. For sea surfaces, however, the flight experiments by [6, 13] indicate that the third Stokes parameter U of sea surfaces is no more than a few tenths of Kelvin at L-band frequencies. Table 1 provides the theoretical predictions of Stokes parameters for sea surfaces at 1.4 GHz with a two-scale scattering model [17] for $10 \text{ m}\cdot\text{s}^{-1}$ winds at an azimuthal angle of 45° relative to the wind direction. The theoretical predictions for U at near nadir incidence angles agree excellently well with the data shown in [6, 13]. The data [6, 13] and theory suggest that the directional dependence of U of sea surfaces is less than about 0.1 Kelvin. In addition, U is shown to be much weaker than Q for above 15° incidence angles, and hence we can ignore the contributions of U to U_a for off-nadir observations of ocean surfaces.

Under the assumption of azimuthal asymmetry or ignoring the contribution of U , we obtain

$$Q_a = Q \cos 2\Omega \quad (19)$$

$$U_a = -Q \sin 2\Omega \quad (20)$$

For ocean and most terrain surfaces, Q is greater than zero because the vertically polarized radiation has a better transmissivity through the surface than the horizontally polarized radiation due to the presence of Brewster angle for vertical polarization [16]. This allows us

to solve for Ω and Q from the above equations:

$$Q = \sqrt{Q_a^2 + U_a^2} \quad (21)$$

$$\tan 2\Omega = -\frac{U_a}{T_{va} - T_{ha}} \quad (22)$$

Note that the above equations permit ambiguous solutions of Ω with an integer multiples of 180° , but this is not a problem because all the ambiguities produce the same corrections for brightness temperatures.

Hence, measurements of the third Stokes parameter provide information of the ionospheric Faraday rotation angle and allow us to make corrections of vertically and horizontally polarized brightness temperatures for natural surfaces with small U Stokes parameter.

3 Error Analysis

The accuracy of the above algorithm is limited by the instrument measurement errors and the departure of the surfaces from azimuthal symmetry. If we represent the above errors by a quantity ΔU , the retrieved Q and Ω will be

$$Q' = \sqrt{Q_a^2 + (U_a + \Delta U)^2} \quad (23)$$

$$\tan 2\Omega' = -\frac{U_a + \Delta U}{T_{va} - T_{ha}} \quad (24)$$

Figure 2 plots the error ($\Delta\Omega = \Omega' - \Omega$) of the Faraday rotation angle estimated from the third Stokes parameter. The upper panel illustrates the error versus Ω for a fixed $Q = 40$ Kelvin and a range of ΔU . It is shown that the accuracy of Ω' can be better than 0.2° for $\Delta U \leq 0.2$ Kelvin. The bottom panel plots the accuracy of Ω' for a fixed value of $\Delta U = 0.2$ Kelvin. A larger second Stokes parameter Q clearly produces a more accurate estimate of Ω . This is expected because U is more sensitive to the Faraday rotation for larger Q .

If the estimated Faraday rotation angle Ω' is used to make corrections of the brightness temperature, the residual error in the horizontally polarized brightness temperature is illustrated in Figure 3. The upper panel indicates that the residual error increases with Ω and is less than 0.1 Kelvin for $\Delta U = 0.2$ Kelvin at up to 30° Faraday rotation angle.

The lower panel in Figure 3 plotting the residual error versus Q demonstrates that the accuracy of the brightness temperature corrections is fairly insensitive to the second Stokes parameter. This can be understood by examining Eqs. (23) and (24). If we assume that Q is much greater than ΔU , Q' can be approximated by

$$Q' \approx Q - \Delta U \sin 2\Omega \quad (25)$$

Therefore the errors of the retrieved brightness temperatures will be

$$\Delta T_h = T'_h - T_h = -(T'_v - T_v) \approx \frac{\Delta U \sin 2\Omega}{2} \quad (26)$$

Here T'_v and T'_h are the corrected brightness temperatures. It appears that the residual brightness temperature errors are proportional to ΔU and are insensitive to Q . This implies that the accuracy of using the third Stokes parameter for brightness temperature corrections is limited by the measurement and modeling accuracy of U . Furthermore because Q is related to the terrain types, a small sensitivity to Q suggests that this technique could provide a uniform brightness temperature correction for a large range of surface conditions.

The estimates of Faraday rotation angle can also be used to infer the ionospheric TEC through Eq. (4) and consequently the ionospheric differential delay. Eq. (5) suggests that N_f can be estimated with an accuracy of about 0.5 TECU for $\Delta\Omega = 0.2^\circ$ and $f = 1.4$ GHz. Therefore, a space-based L-band polarimetric radiometer can potentially provide global ionospheric TEC measurements with an accuracy of about 1 TECU. Based on the relationship of ionospheric TEC and path delay described in [15], 1 TECU produces about 0.35 ns (10.5 cm one-way) differential delay at L-band. Because the ionospheric differential delay is proportional to $1/f^2$, the accuracy will approach 3.5 ps (1 mm one-way) at Ku-band (14 GHz). This suggests that this technique could provide accurate estimates of ionospheric differential delay for sea surface height measurements with spaceborne altimeters.

4 Conceptual Design

A conceptual design (Fig. 4) for the measurement of the third Stokes parameter is proposed. This design is applicable to both real aperture and aperture synthesis radiometers. In this design, the surface radiation from the antenna is split into vertical and horizontal polarizations by the ortho-mode transducer (OMT). The signals from both polarization channels are amplified and then split into two channels for each polarization. A microwave-hybrid then adds and subtracts the vertically and horizontally polarized signals to produce $\pm 45^\circ$ linear polarizations.

For real-aperture radiometers, the power of signals from the vertical, horizontal, 45° -linear and -45° linear polarizations is detected to obtain T_v , T_h , T_{45} and T_{-45} , respectively. The third Stokes parameter U can be derived from any one of the following equations:

$$U = T_{45} - T_{-45} \quad (27)$$

$$U = 2T_{45} - T_v - T_h \quad (28)$$

$$U = T_v + T_h - 2T_{-45} \quad (29)$$

These equations have been implemented and tested for the Jet Propulsion Laboratory aircraft polarimetric radiometers [8]. The second and third equations require either T_{45} or T_{-45} measurements to produce the third Stokes parameter, and hence can probably simplify the design of microwave hybrid and uses only three power detectors, instead of four.

The design indicated above is also applicable to aperture-synthesis radiometers [10, 11, 12], which take the output from a distributed set of antennas and correlate the signals to form the image. For this type of radiometers, the microwave scheme proposed here has to be implemented for each antenna and associated front-end receiver. The back-end correlators can then switch between these polarization channels to form the images of T_v , T_h , T_{45} and T_{-45} . The third Stokes parameter for each pixel in the image can then be synthesized from the equations described above.

5 Summary

This article suggests the use of polarimetric passive microwave measurements for the estimates of ionospheric Faraday rotation. Applications can be considered for microwave radiometry of ocean surface salinity of soil moisture and altimetry of sea surface height. The analysis shows that this technique can provide accurate estimates of Faraday rotation angle and is effective for the corrections of radiometric errors due to Faraday rotation. With measurement errors for the third Stokes parameter in the order of 0.1-0.2 Kelvin, it is shown that the residual errors in the corrected brightness temperatures can be reduced to less than 0.1 Kelvin. This makes it feasible to use low-frequency microwave waves to remotely sense ocean salinity and soil moisture from space for day and night satellite passes. A conceptual design for hardware implementation is described. It is suggested that this technique can be implemented for both real-aperture and aperture-synthesis radiometers.

The key assumption of this technique is that the surfaces illuminated by the antenna have to be nearly azimuthally symmetric with negligible third Stokes parameter. This assumption should be valid for most natural terrain surfaces, but it is clearly desirable to know how well this technique will work for agricultural plow fields in particular. Therefore, aircraft and ground-based polarimetric measurements are suggested to determine the amplitude of the third Stokes parameter of microwave radiation from agricultural fields.

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List of Tables

1	Sea surface brightness temperatures at 1.4 GHz predicted by a polarimetric two-scale scattering model [17] at $10 \text{ m}\cdot\text{s}^{-1}$ wind speed and 45° azimuth angle relative to the wind direction.	14
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Incidence Angle (degree)	T_v (K)	T_h (K)	Q (K)	U (K)
10	95.1	92.9	2.2	-0.12
15	96.6	91.5	5.1	-0.11
20	98.6	89.7	9.0	-0.11
30	105.1	84.4	20.7	-0.11
40	115.2	77.0	38.2	-0.10
50	130.4	67.6	62.8	-0.09

Table 1: Sea surface brightness temperatures at 1.4 GHz predicted by a polarimetric two-scale scattering model [17] at $10 \text{ m}\cdot\text{s}^{-1}$ wind speed and 45° azimuth angle relative to the wind direction.

List of Figures

1	The effects of Faraday rotation on the Stokes parameters of microwave radiation from earth surfaces. The upper panel plots the change of horizontally polarized brightness temperatures for five different values of Q versus the Faraday rotation angle. The bottom panel plots the third Stokes parameter resulting from the Faraday rotation	16
2	Accuracy of the Faraday rotation angle estimated from the third Stokes parameter measurements. The upper illustrates the accuracy for $Q = 40$ Kelvin and five different ΔU values. The lower panel plots the errors for $\Delta U = 0.2$ Kelvin and five different values of Q	17
3	Estimated accuracy of brightness temperatures corrected with the Faraday rotation angle estimated from the third Stokes parameter measurements. The upper illustrates the residual error of T_h for $Q = 40$ Kelvin which approximately corresponds the sea surface radiation at 40° incidence and 1.4 GHz. The lower panel plots the residual errors versus Q for $\Delta U = 0.2$ Kelvin and three different values of Ω	18
4	A conceptual design for Stokes parameter measurements applicable to real aperture and aperture synthesis radiometers.	19

BRIGHTNESS TEMPERATURE ERROR VERSUS FARADAY ROTATION

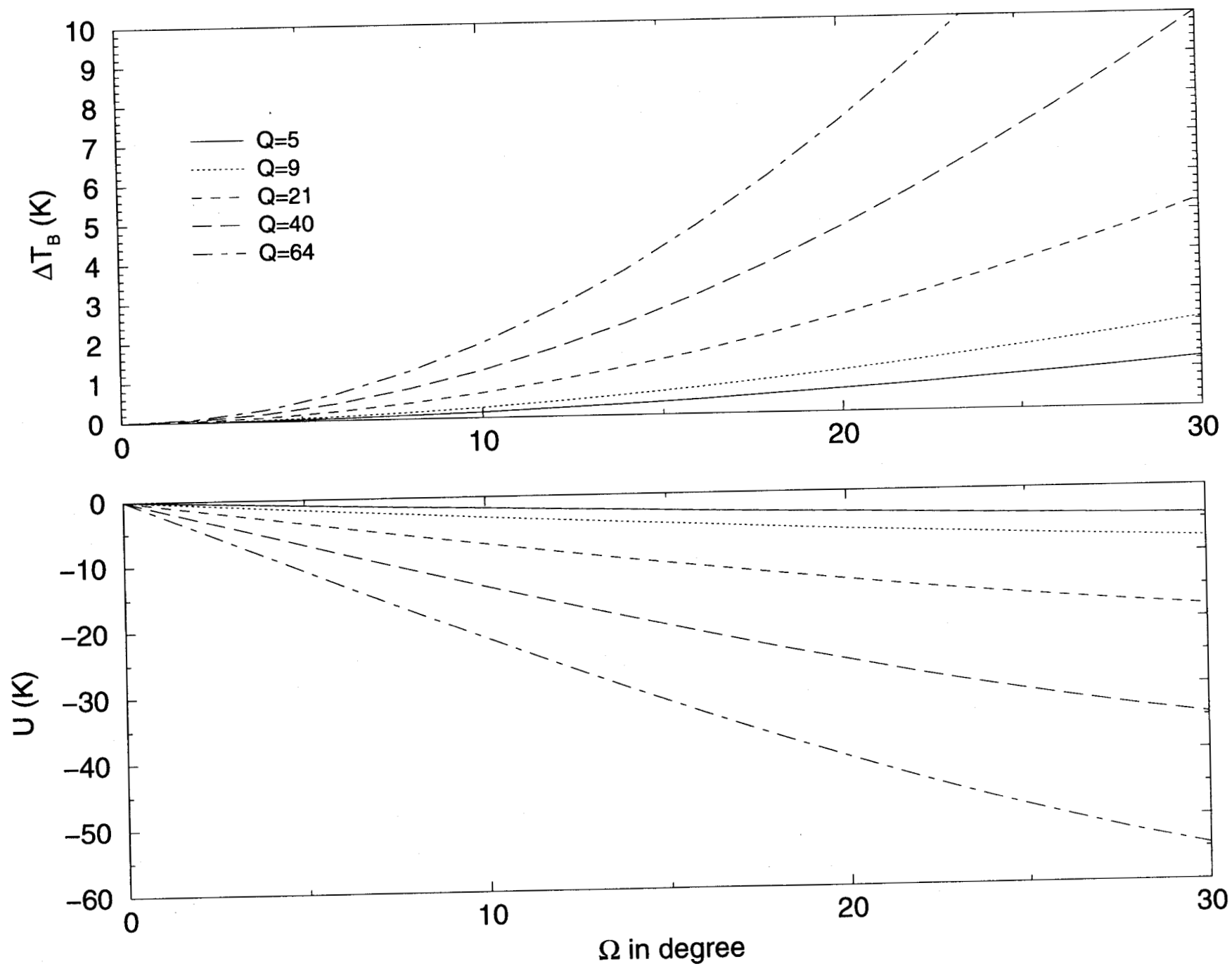
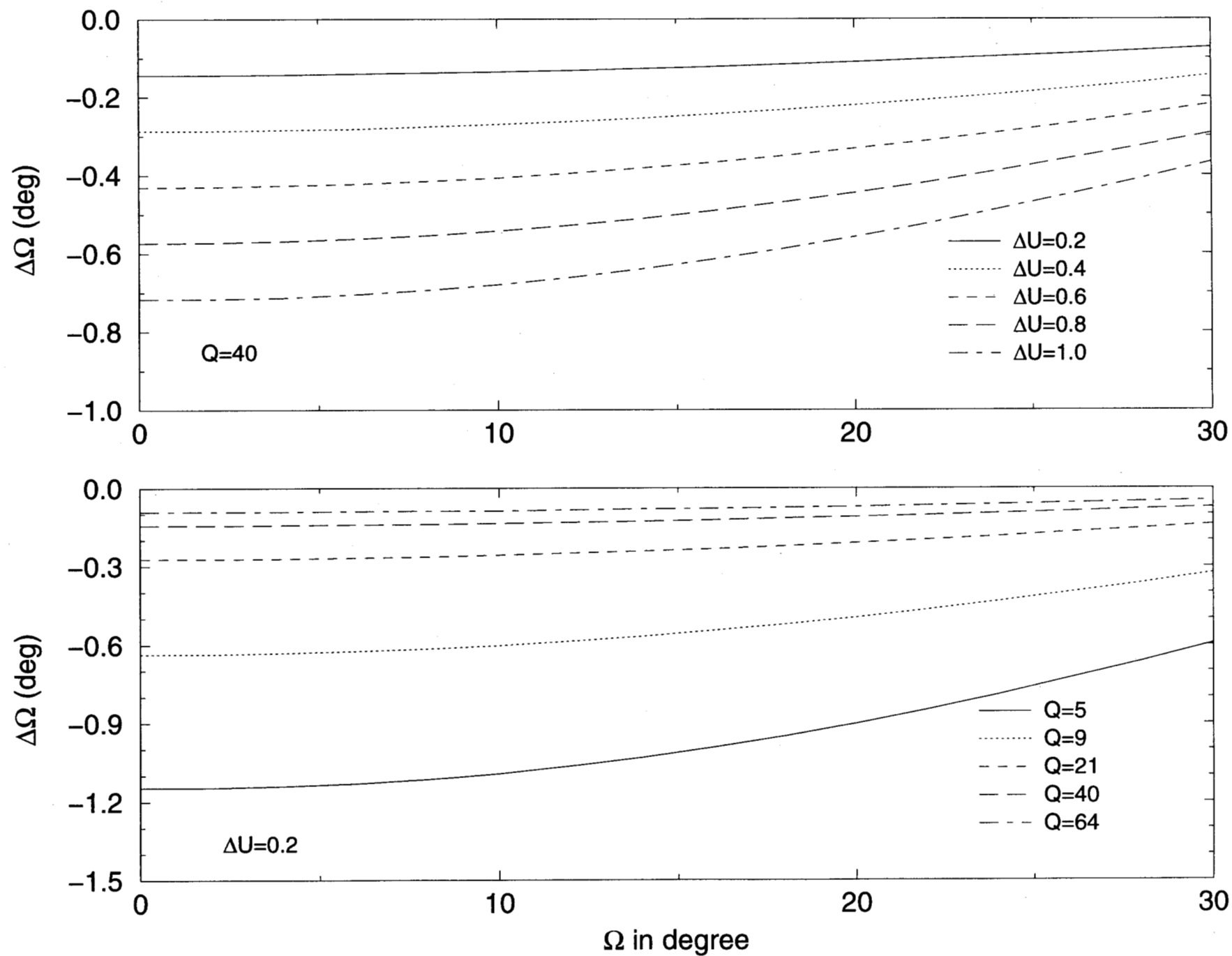


Fig. 1

ACCURACY ESTIMATE OF FARADAY ROTATION ANGLE



ERROR ESTIMATES OF BRIGHTNESS TEMPERATURE CORRECTIONS

